

Differential Equations (MAS118) Problem Sheet 4

Problem 10

Determine the general solution of the following homogeneous linear differential equations. Fix for each solution the constant of integration according to the given initial condition.

a) $y' = -xy$, $y(0) = -2$ (4 points)

b) $y' = x \cos(x)y$, $y(0) = 1$ (4 points)

c) $y' = -y/(1+x)$, $y(0) = -1$ (4 points)

d) $y' = y/(4-x^2)$, $y(0) = 0$ (4 points)

e) $y' = y/(x^2 + 2x + 2)$, $y(0) = 2$ (4 points)

Problem 11

Determine the general solution of the following inhomogeneous linear differential equations. Fix for each solution the constant of integration according to the given initial condition.

a) $y' = 3y + 5$, $y(0) = -2$ (12 points)

b) $y' = -2xy + 2x$, $y(0) = 1$ (12 points)

c) $y' = 3x^2y/(1+x^3) + x^2 + x^5$, $y(0) = -1$ (12 points)

d) $y' = y + 2xe^{2x}$, $y(0) = 3$ (12 points)

e) $y' = -y \tan(x) + \cos(x)$, $y(0) = 2$ (12 points)

Problem 12

Consider the differential equation

$$y' = y - xy^3$$

a) Use the substitution $z(x) = 1/(y(x))^2$ to rewrite the differential equation in terms of the new dependent variable z . (10 points)

b) Solve the linear inhomogeneous differential equation for z and determine the general solution $y(x)$ of the original differential equation. (10 points)

Problem D

Solve the linear inhomogeneous differential equation

$$y' = -y/x + 1$$

with initial condition $y(0) = -1$. Any comments?

(20 bonus points)